Coming to 'Know' Mathematics through 'Acting, Talking and Doing' Mathematics

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This paper adds to a discussion initiated by Askew (2007) about two contrasting views of scaffolding; as a 'tool for results' and a 'tool-and-result'. The study took place in four primary classrooms within a low socioeconomic setting. Two classroom episodes drawn from one of the teacher's classroom, illustrate the two different perspectives of scaffolding. These are presented and the learning that evolved from each episode is discussed. The paper illustrates that when scaffolding was used as a 'tool for results' the learning was restricted but when the students were scaffolded within a tool-and-result perspective the mathematical knowledge and ways of doing and talking mathematics were generative.

In a recent PME paper Askew (Askew, 2007) drew our attention to a need to re-consider the viability of Vygotskian notions of scaffolding within current mathematics classrooms. In this paper, Askew challenged the usefulness of mathematics educators adopting the metaphorical view of scaffolding as a 'tool for results'. Askew linked this view of scaffolding as a 'tool for results' to the recent reform measures introduced in Britain. Within these reform measures, policy makers specify detailed learning outcomes and activity for those involved in teaching mathematics. In recent years New Zealand has followed a similar route with the introduction of a 'Numeracy Development Project' (Ministry of Education, 2004a). Similar to the British model, a recently introduced New Zealand Numeracy project has predetermined learning outcomes and a detailed script for teachers to use. Implicitly suggested within this New Zealand model, is the idea of scaffolding as a mediating tool that will give results—student acquisition of mathematical knowledge and strategies through teacher led instruction. In contrast, Askew building on Vygotskian theories interpreted in the work of Newman and Holzman (1997), proposed that mathematics educators should adopt an alternative stance to scaffolding; one in which scaffolding is considered as a 'tool-and-result'. Within this contrasting perspective scaffolding as the mediating tool, is as much a part of the learning as is what is learnt. This paper explores the concept of scaffolding, as it is used to support student learning in two classroom episodes, and the student learning which emerges as a result. The aim of the paper is to illustrate in two contrasting episodes what happens to students 'talking and doing mathematics' like mathematicians when scaffolding as the mediating tool fits within a metaphorical view of it as either a 'tool for results', or as a 'tool-and-result'.

According to Vygotskian thinking, conceptual reasoning developed in mathematics classrooms is a result of interaction between everyday spontaneous concepts and scientific concepts. Scientific concepts involve higher order thinking, which are used as students engage in more proficient forms of 'doing and talking' mathematics. Vygotsky (1986) maintained that, "the process of acquiring scientific concepts reaches far beyond the immediate experience of the child" (p. 161). Although his work was not within the schooling system he suggested that school was the cultural medium, with dialogue the tool that mediated transformation of everyday spontaneous concepts to scientific concepts. Vygotsky's suggestion was not, however, that scientific concepts are separate from spontaneous concepts, nor the act or practice of their development separate from their result. Rather, Vygotsky argued that they were an integral part of both the process and the

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outcomes. Askew (2007) illustrated what Vygotsky described in his professional development work with teachers and students. Askew persuasively illustrated that the performance and the creation of mathematical objectives is as much a priority for learning, as is the knowledge learnt. Through the construction of a learning environment in which students were both encouraged and required to talk mathematically, Askew illustrated how the immediate importance of the lesson learning outcome gave way to the bigger priority—that the students learnt to talk and act as mathematicians. Furthermore, through the specific scaffolding they received they learnt that they had the choice to continue to think, talk, and act, like mathematicians when doing mathematical activity.

Scaffolded Mathematical Discourse within Zones of Proximal Development

Whilst the exact nature of how external articulation becomes thought has been extensively debated (Sawyer, 2006), sociocultural theorists are united in their belief that collaboration and conversation are crucial to the transformation of external communication to internal thought. They suggest that this occurs as students and teachers interact in co-constructed zones of proximal development. The zone of proximal development has been widely interpreted as a region of achievement between what can be realised by individuals acting alone and what can be realised in partnership with others (Goos, Galbraith, & Renshaw, 1999). Traditional applications of zones of proximal development were used primarily to consider and explain how novices are scaffolded by experts in mathematical activity. Taking the view Askew (2007) proposed—the tool-and-results perspective—widens the frame and supports ways to consider the scaffolded learning, which occurs when levels of competence are more evenly distributed across the members of the zone of proximal development. Mathematical learning in this form occurs during mutual engagement in collective reasoning discourse and activity (Mercer, 2000). Lerman (2001) describes collective participation in mathematical discourse and reasoning practices as pulling all participants forward into their zones of proximal development which he terms a symbolic space—"an ever-emergent phenomenon triggered, where it happens, by the participants catching each other's activity" (p. 103).

Defining the zone of proximal development as a symbolic space provides a useful means to explain how participants in classrooms mutually appropriate eachothers' actions and goals. In doing so, they are required to mutually engage and inquire into the perspectives taken by other participants. In such learning environments teachers, too, are pulled into the zone of proximal development and are required to understand from the perspective of their students, their reasoning and attitudes (Goos, 2004). Mercer (2000) termed this process of inquiry into each other's reasoning "interthinking" (p. 141). During interthinking Mercer outlined how the variable contributions of participants create a need for continual renegotiation, and reconstitution of the zone of proximal development. In the extended discourse the contributions are critiqued, refined, extended, challenged, synthesised and integrated within a collective view. At the same time, all members' mathematical reasoning is scaffolded beyond a level they could achieve alone.

The construct of interthinking—pulling participants into a shared communicative space—extends the view of scaffolding and the zone of proximal development. It supports consideration of the learning potential for pairs or groups of students working together with others of similar levels of expertise in egalitarian relationships (Goos, 2004; Goos et al., 1999). The partial knowledge and skills that group members contribute, support and deepen collective understanding. Opportunities are also provided for the group to encounter mathematical situations, which involve erroneous thinking, doubt, confusion and uncertainty. Importantly, constructing a collective view is not always premised immediately on consensus. Dissension can also be a catalyst for progress either during, or after, a collaborative session (Mercer, 2000) and to reach consensus, negotiation requires participants to engage in exploration and speculation of mathematical reasoning—an activity, which approximates the actual practices of mathematicians. Such scaffolded activity inducts students into more disciplined reasoning practices. The "lived culture of the classroom becomes in itself, a challenge for students to move beyond their established competencies" (Goos et al., 1999, p. 97) to become more autonomous participants in mathematical activity and talk.

Research Design

This paper reports on episodes drawn from a larger classroom-based design research study (Hunter, 2007a). The study was conducted at a New Zealand urban primary school and involved four teachers and 120 Year 4-8 students (8-11 year olds). The students were from low socio-economic backgrounds and were pre-dominantly of Pasifika or New Zealand Maori ethnic origin. The teachers had completed a professional development programme in the New Zealand Numeracy Development Project (Ministry of Education, 2004a). They reported at the start of the study that their students had poor mathematical achievement levels. They also considered that asking their Maori and Pasifika students to explain their reasoning, or challenge the reasoning of others, had considerable difficulties both socially and culturally for this grouping of students. At the conclusion of the study the students were achieving at a level, which placed them at a sound level of achievement.

A year-long partnership between the researcher and teachers using a design research approach supported the design and use of a 'Participation and Communication Framework' and a 'Framework of Questions and Prompts'. The 'Participation and Communication Framework' was designed as an organising tool to assist the teachers to scaffold students' use of proficient mathematical practices within reasoned inquiry and argumentation. The 'Framework of Questions and Prompts' was a tool co-constructed during the study to deepen student questioning and inquiry. More detail (Hunter, 2006, 2007a, 2007b, 2008) of the Frameworks and how these scaffolded teacher change and student change can be found in previous PME and MERGA papers. Student development of mathematical practices is not the focus of this paper. The focus is on how they were inducted into the discourse of inquiry and argumentation, and the ways in which this influenced how they interacted in zones of proximal development. For this paper, two episodes were selected to illustrate how scaffolding was used within the classroom context by one of the teachers and how it influenced the students' engagement in 'talking and doing' mathematics.

Results and Discussion

In the initial stages of the study the four teachers in the research closely adhered to the structured lessons provided in the New Zealand Numeracy Development Project material (Ministry of Education, 2004a). At this early stage in the study the scripted lessons were often followed word for word from the curriculum material provided by the developers. This section illustrates what happens when the scripted lessons scaffold what the teacher does and how they are used as a 'tool for results'.

Scaffolding as a 'Tool for Results'.

The teacher began the lesson by stating a learning intention that signalled what he expected the outcome of the lesson on fractions should be. He began by reading:

Teacher: So what we are doing today is that we are learning to find fractions of a set.

He continued reading the script (See Ministry of Education, 2004b, p. 7).

Teacher: Here is a farm [draws a cut in two fields on a piece of paper]. The farmer uses an electric fence to make her farm into two paddocks. She has ten animals. Hinemoa you count out ten of those animals [Hinemoa counts out one by one ten plastic animals]. She wants to put one-half of the animals in one paddock and one-half in the other. How many animals do you think will be in each paddock?

Jo: I already know five because five and five are ten.

Without acknowledging Jo's interjection he directed the students:

Teacher: We all need to take ten animals and share them into the two paddocks in our groups. You need to turn to your partners because you are working together in your groups of three and talk about what you are doing.

In their groups, the students took the ten animals, which they counted out one by one into two groups. The only discussion was about counting out the animals one by one, then counting the two sets and agreeing that there are five animals in each set. They each took a turn to do this. The teacher watched and when he observed that all the groups had completed the task he returned to the script:

Teacher: Could we have worked out the number of animals in each paddock without sharing them out?

Jenny: Yes we could say five plus five is ten.

Teacher: [The teacher picks up two groups of five and shows the students] Yes you can use your doubles and say five and five.

He continued the lesson posing similar problems and directing the students to use materials and share out the animals in order to find the answer to the problem. Each time the students completed the task he asked one student to explain what was done:

Hone: We had fourteen bears so one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen so seven and seven are fourteen.

The lesson was orderly and the students responded almost as if they were performing, playing a game of turn-taking, sharing out the animals and saying the matching script related to double addition modelled by their teacher. The teacher had followed the script closely. At the conclusion of this lesson he stated that the students now knew how to find a fraction of a set and were ready to move to the next lesson outlined in the curriculum material.

Considering this lesson it appears that both sets of individuals have been scaffolded to play specific roles. The teacher used the script to play out a role in which he can address a specific and narrow learning outcome, which he has detailed at the start of the lesson. The role he took was to show and tell. The students in turn adopted the role he cast them in and played out their role to acquire the specific piece of mathematical knowledge. They were 'talking and doing' mathematics but the question is, "What knowledge of themselves as mathematicians were they developing?" Moreover, "What were they learning about talking and doing mathematics?"

The following section contrasts the preceding lesson episode, which occurred in the first week of the study with a mathematics lesson which took place towards the end of the study.

Scaffolding as a 'Tool and Results'.

As outlined in the Research Design section, extensive scaffolding was provided by the teacher to support students to use a range of proficient mathematical practices including reasoned mathematical explanations, justification and generalisations. Scaffolding was also used to support students to develop a repertoire of questions and prompts to use to inquire into the sense-making of others. In addition, the teachers also paid specific attention to establishing group norms to ensure interthinking occurred. In relationship to the New Zealand Numeracy Development Project, the teacher continued to draw on the curriculum material to provide guidance for his lessons. But then now he no longer followed the script and he wrote problems, which better matched the interests of his students.

In this lesson the teacher wanted the students to explore the strategy of partitioning but he had selected numbers, which support emergence of multiple ways of reasoning towards a solution strategy. The lesson consisted of two components; small group problem solving and then a large group discussion. This episode describes the first section of the lesson in which the students had been placed in groups of three and without teacher-led discussion they were given a problem[25] and asked to discuss and develop a number of solution strategies.

Saawan: What about five times 700 and then…

Hine: Five times fifty, and then five times six.

Sonny: Hey mine's the same but mine's starting from the six, fifty, and then seven hundred. Hey all our ways are the same, well kind of, because you can start both ways.

Saawan: Well let's see if that right…so you say we can start both ways, yeah that's cool it works.

The students began immediately to work together, interthinking, and they constructed a solution strategy using the distributive property. They continued to discuss and explore whether the order of how the factors were distributed affected the solution as they recorded them in the different ways. As Sonny studied the recordings he introduced the group to an alternative idea. This strategy was one that drew on distributing the factor of five rather than the factor of 756:

Sonny: I have just thought and I know another way. Can you do seven hundred and fifty six times two and then plus it so the times two becomes…becomes times four…equals…

Saawan: What? Let's write it down.

Sonny was playing with the idea of the generalisation the group had collectively constructed. He introduced it as he thought out loud and Saawan's answer indicated that although he had not yet made sense of what Sonny was saying he was open to the new contribution. Sonny showed that his thinking was still being formed when Hine recorded it vertically as 756 + 756 and he told her:

Sonny: No times two is easier.

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[25] Bart Simpson had five different coloured marbles. He had 756 of each colour and Lisa wants to know how many he has altogether. Can you help him tell Lisa how many he has? Lisa might challenge him to prove he has more than her so can you work out some different strategies he could use?

Saawan followed Sonny's reasoning closely and his argument indicated that he was making a link to their previous reasoning. He then extended his reasoning and that of his peers when he argued that multiplication was repeated addition:

Saawan: Times two yeah but doing it that way is the same way really, you can say it as a plus because that's the same as times like before when we went the other two ways not just one way.

Hine, listening to the exchange crossed out the recording, replacing it with 756 x 2. Then Sonny continued with the new thinking as Hine and Saawan tracked closely and examined the reasoning section by section:

Sonny: Seven hundred and fifty six times two equals one thousand six hundred and twelve…

Hine: Wait, one thousand… [Lapses into silence as she records 700 x 2 then writes 50 x 2 and 6 x 5].

All three students examined the recordings and checked the total. Then Saawan took the pen, from Hine and he recorded 1512 x 2 as he continued to explain:

Sawaan: And then we times, no we add them together then times it by two and add seven hundred and fifty six on to it [Records 3780].

Hine: But hang on how did we get that?

Sonny: [Directs her attention to the recording as he explains] By timsing this by two, and this by two, and then adding.

Hine: [Nods her head] Yeah I get it now.

The teacher had been sitting silently listening and observing the interaction. Then he observed Hine's continuing uncertainty and so he prompted her to question, emphasising that she needed to do so until she had complete understanding

Teacher: You look like you are still a bit puzzled. Look at what he has explained and if you need to, ask more questions. Make sure you are convinced that it works. Think about a good question and ask it.

Hine: Why did you times one thousand five hundred and twelve by two?

Saawan: Because it's like…because then when we times that by two [he points at the second two] it is like that will be like four and then we only have to add seven hundred and fifty six. It's just doubling.

The teacher's prompt for further questioning left the mathematical agency with the students. After closely listening to the student provided explanation he then pressed them to further explore the reasoning:

Teacher: By adding this [He points at + 756] what's another way of saying that because I think maybe that…how could you say it differently instead of saying adding seven hundred and fifty six?

Now Sonny and Hine indicated that the reasoning Saawan introduced had become integrated within their collective understandings:

Sonny: You could multiply it by one…

Hine: Okay, I get it now so multiply by one yeah so when we times two, times two, times one because the whole thing is seven hundred and fifty six times five, so times five yeah, [she laughs then refers to the context of the problem] huh that's a good one Lisa better understand from Bart.

In this second lesson scaffolding took a different form from that reported in the first lesson. Scaffolding had become a tool, which mediated the mutual engagement of all participants in the collective reasoning. The use of *problem solving* groups where mathematical expertise was more evenly distributed across the members changed their interactions. This resulted in each individual's role emerging and changing minute by minute in the discussion, as they were pulled into a shared communicative space. The different contributions scaffolded the group members being extended beyond their own capabilities. Importantly, the mathematical understanding they were developing was of equal importance to what they were learning about acting as mathematicians and 'talking and doing' mathematics.

Conclusions and Implications

The paper sought to explore and examine scaffolding used in two different ways in classroom episodes, and the learning, which emerged as a result. The paper illustrated that when scaffolding is used as a tightly controlled tool within what Askew (2007) describes as a "technical-rationalist view of teaching and learning" (p. 239) the roles the teacher and the students hold and the mathematical talk they use and the knowledge they develop is limited. Likewise, the students' learning to 'talk and do' mathematics in ways mathematicians do, are restricted. However, when scaffolding is used within a widened dimension that affirms both the importance of the construction of mathematical knowledge and the manner in which it is constructed, the learning potential for all participants is enhanced.

This paper confirms the results in Askew's (2007) PME paper but extends these results to show the learning potential available when teachers scaffold students to work together to construct a collective mathematical view within zones of proximal development. As other researchers (Goos, 2004; Goos et al., 1999; Lerman, 2001; Mercer, 2000) have illustrated, the act of interthinking and developing a collective view was a key factor which scaffolded how these students learnt to talk and do mathematics. Of importance too, was careful teacher preparation, which drew on the New Zealand Numeracy Project as a tool for classroom activities rather than a rigidly followed formula. The use of grouping and the careful selection of numbers allowed the lesson to unfold and the students to improvise and play with the numbers, in a form of mathematics, which was generative.

Implications of this study suggest the need for mathematics educators to consider not only the importance of the development of mathematical knowledge but also how it is constructed. In this form scaffolding needs to be metaphorically viewed as a 'tool-and-result'. National projects such as the New Zealand Numeracy Project (Ministry of Education, 2004) have an important place as a professional development tool but teachers need to develop their own script rather than use the materials rigidly.

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